Integers and Properties

(a modified CMP unit to better fit MN standards)

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Number & Operation unit for 7th graders

Special note: The materials in this unit are copyright from CMP, 2nd edition, Samples and Populations unit. Most districts have transitioned to CMP3, the newest edition of the program.

In order to fully teach this unit, teachers would need access to the student pages of the program which can be found in the BSU Math Curriculum Library. Alternatively, CMP 2nd edition resources can be found quite inexpensively online through sites like Amazon.com.

In the interest of trying to stay within the fair use of copyrighted materials, the student pages have been excluded from this lesson set.

Works Cited:

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2009). *Accentuate the Negative* (2nd ed., Connected Mathematics). Upper Saddle River, NJ: Pearson.

These 15 days address three Number& Operation and Algebra standards in 7th grade, with a focus on developing models and algorithms to multiply and divide positive and negative rational numbers and then apply that knowledge to the order of operations. The 15 days are composed of 2 different problem solving investigations. This table provides the links between which standards are addressed in which investigations and the overall goal of the lesson.

Since a majority of our materials are gathered from the CMP, 2nd edition, program, this table also provides the necessary reference for how the lesson objectives related to the MN standards.

Page	Days	MN Standard	Lesson		Lesson Objective	Sample MCA Question
pg 3-6	**0**	**Pretest**				
pg 7-10	**1**	**7.1.2.1.** Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents. **7.1.2.2.** Use real-world contexts and the inverse relationship between addition and subtraction to explain why the procedures of arithmetic with negative rational numbers make sense.	Inv. 3	Intro to Models	Apply knowledge of bar models and fact triangles to modeling multiplying and dividing positive and negative rational numbers.	See 7th Grade MCA item sampler, #1 & 2
pg 11	**2**				Use chips and number lines to understand the basic process of multiplication and division of integers.	
pg 12-13	**3**			3.1	Develop an algorithm for multiplying integers.	
pg 14	**4**			3.2	Examine patterns to confirm algorithm for multiplication.	
pg 15-16	**5**			3.3	Develop an algorithm for dividing integers.	
pg 17	**6**			Practice	Apply knowledge of models to solve division problems with positive and negative rational numbers.	
pg 18-19	**7**					
pg 20-21	**8**			3.4 + practice	Practice fluency with operations for positive and negative rational numbers.	
pg 22-23	**9**	**7.2.3.1** Use properties of algebra to generate equivalent numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws.	Inv. 4	4.1	Explore the use of order of operations.	See 7th Grade MCA item sampler, #21
pg 24-25	**10-11**			4.2	Develop the distributive property of multiplication over addition.	
pg 26-27	**12**			4.3	Develop the distribute property of multiplication over subtraction	
pg 28-29	**13**			Practice	Multiply and divide integers.	
pg 29-35	**14-15**			Dealing Down	Understand distributive and commutative properties and know which operations they hold true for. Apply order of operations to make computational sequences clear.	
pg 36-38		**Postest**				

Name:_____

Pretest – Number & Operation Unit

1.

Find these products and quotients.

a. $13 \times {}^{-}7 =$

b. ${}^{-}8 \times {}^{-}20 =$

c. $99 \div {}^{-}3 =$

d. $\frac{-36}{-12} =$

e. $0 \div 18 =$

f. $\frac{1}{3} \times \frac{-5}{7} =$

2.

A bakery bought 225 pounds of baking powder. It used 6.8 pounds per day. How much did the bakery have left after three days? Write number sentences to show your work.

3.

Ray is in debt $32 right now. He had owed more, but he has been paying $6 a month on his debt for the last five months.

a. How much was Ray in debt five months ago?

b. At his present rate, how much longer will it take Ray to pay off his debt? Explain your reasoning.

4. For the 3 problems below, model each situation and then choose the correct result.

$^{-}8 \times {}^{-}29 =$

A. 232 **B.** $^{-}232$ **C.** 211 **D.** $^{-}211$

$^{-}23 \div 0.5 =$

F. $^{-}11.5$ **G.** $^{-}46$ **H.** 11.5 **J.** 46

$45 - 23 \times 2 + 1 =$

A. 43 **B.** $^{-}22$ **C.** 0 **D.** 68

5. Model or write a story for a problem that is answered by finding the value of n.

-4n = -24

6.

Chris and Elizabeth are making a version of the Integer Product Game in which players need three products in a row to win. What six factors do they need for their game?

Chris and Elizabeth's Product Game

4	–4	6	–6
9	–9	10	–10
15	–15	25	–25

Factors:

7.

Malique wants to take four of her friends to a movie. She knows it is $5.50 for a ticket and $3.25 for popcorn.

a. How much will it cost if she pays for the movie and popcorn for all five people?

b. Write a number sentence to show how you computed the total cost.

c. Find a different way to calculate the total cost and show it in a number sentence.

8.

Given the expression $6 \times (14 - 7)$, find the expression that is NOT equivalent.

F. $(6 \times 14) - 7$

G. 6×7

H. $(6 \times 14) - (6 \times 7)$

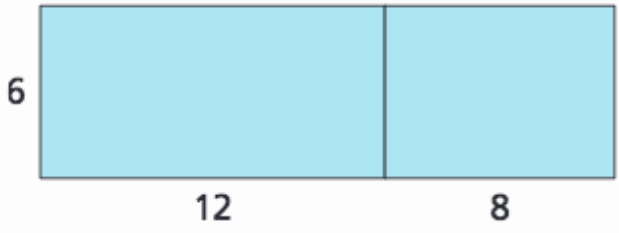
J. $(6 \times 14) + (6 \times -7)$

9.

Write equivalent expressions to show two different ways to find the area of each rectangle. Use the ideas of the Distributive Property.

a.

b.



10. Place a < or > or + sign to make each pair of equations a true statement.

36. $-23 \square -45$

37. $-23 + 10 \square -45 + 10$

38. $-23 - 10 \square -45 - 10$

39. $-23 \times 10 \square -45 \times 10$

40. $-23 \times (-10) \square -45 \times (-10)$

11.

A grocery store receipt shows 5% state tax due on laundry detergent and a flower bouquet.

Laundry Detergent	$7.99	T
Flower Bouquet	$3.99	T

Does it matter whether the tax is calculated on each separate item or the total cost? Explain.

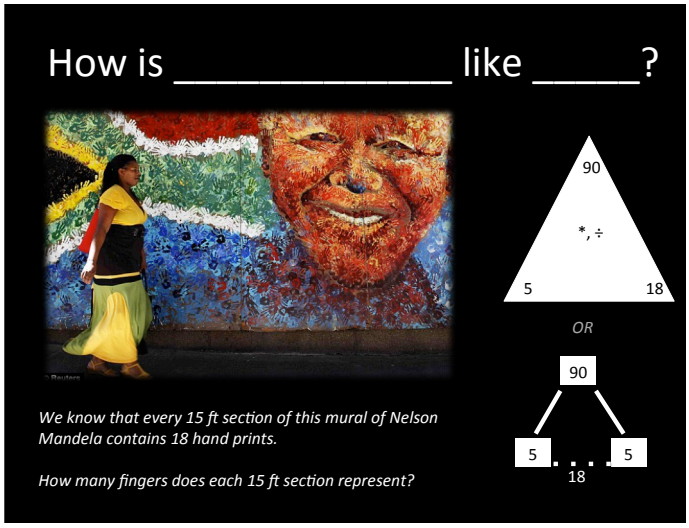
12.

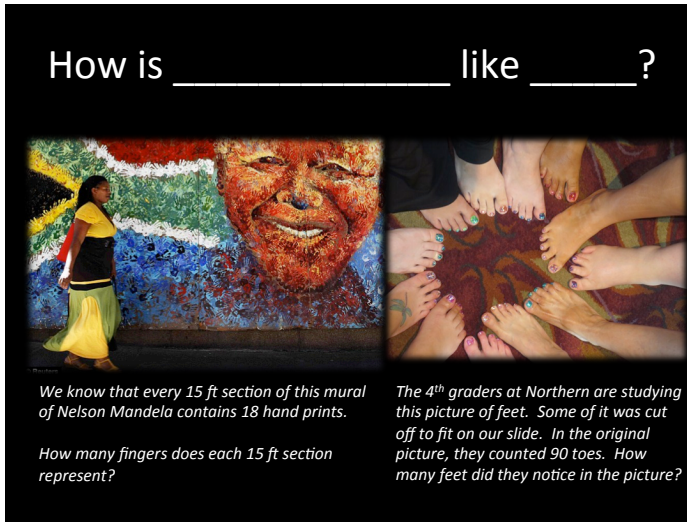
Insert parentheses (or brackets) in each expression where needed to show how to get each result.

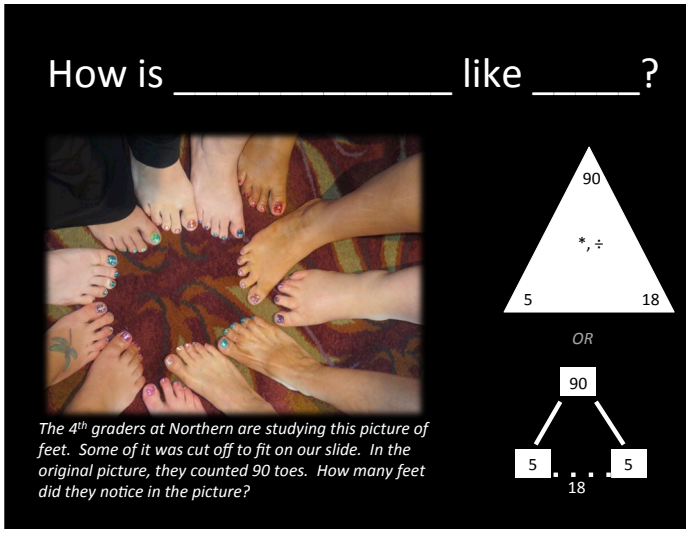
a. $1 + (-3) \times (-4) = 8$ **b.** $1 + (-3) \times (-4) = 13$

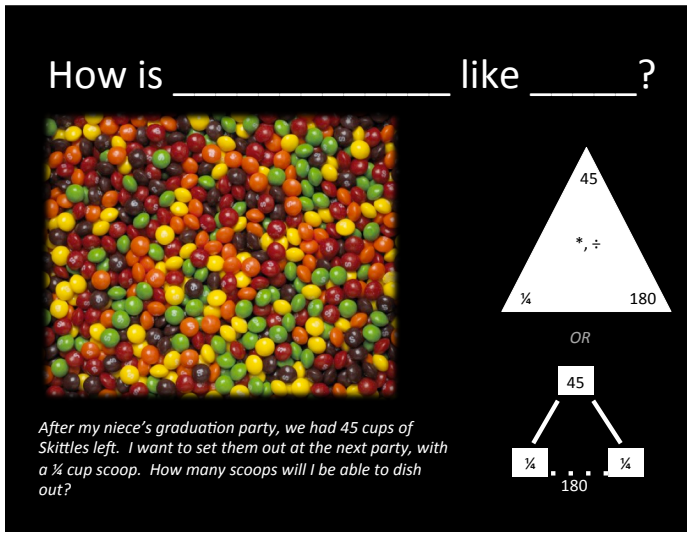
Lesson 1 – Bar Models and Fact Triangles.

The following slides guide the Launch of this lesson. The slides facilitate a discussion about how bar models and fact triangles can help model rational number multiplication and division. The structure of How is ___ like ___ was intentionally used to keep the conversation open enough that students will begin to connect the situations to the models on their own. The slides flow in this order: top left, bottom left, top right, bottom right.



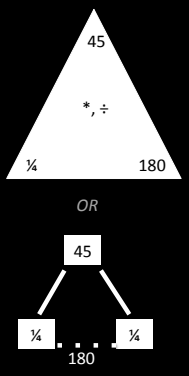






How is _____ like _____?

Pedro counted his quarter collection. He has 180 quarters. How much money does he have? In dollars?



How is _____ like _____?

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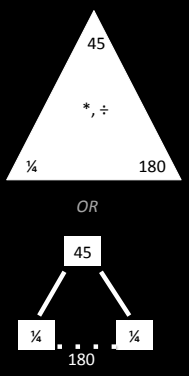
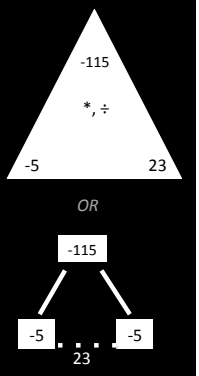


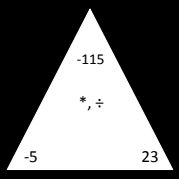
How is _____ like _____?

Mario saved $115 from his law-mowing job this summer. He plans to rent 2 games a week for $5 each week. How many weeks will it be until he is out of money?

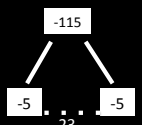


How is _____ like _____?

In Barrow, Alaska, starting on September 22nd, each day they lose 5 minutes of daylight until the winter solstice. How many minutes will they have lost 23 days later, on October 15th?

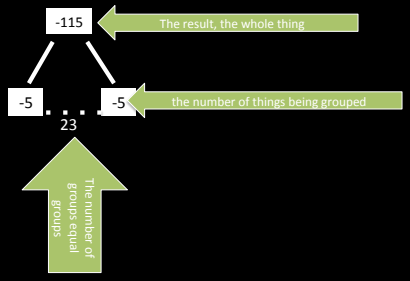


OR



In your journals:

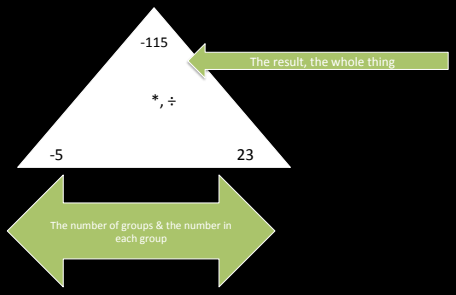
Bar Model for * and ÷



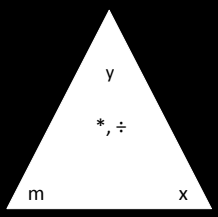
How is _____ like _____?

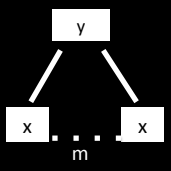
In your journals:

Fact Triangle for * and ÷



How is _____ like _____?





In your journals….

For each problem….

Create a

- Fact triangle
- Bar Model
- Equation or expression

Be prepared to discuss the advantages and disadvantages for each representation during Math Talk.

19. Your teacher purchases 24 pastries for a class celebration, at $2 each. What integer expresses the amount he paid?

20. Temperatures have been falling steadily at 5°F each day. What integer expresses the change in temperature in degrees 7 days from today?

21. A submarine starts at the surface of the Pacific Ocean and descends 60 feet every hour. What integer expresses the submarine's depth in feet after 6 hours?

22. A skydiver falls at approximately 10 meters per second. Write a number sentence to express how many meters he will fall in 40 seconds.

After the students have had time to model and solve 19-22 in 3 different ways, host a SHARE around student solutions for #22.

At the close of that share, surface student ideas about these questions related to modeling problems.

1) Which model helps us see that multiplying and dividing are related to making equal groups?

2) Which model is the most efficient, where you can be accurate and swift?

3) Which model shows us that multiplication and division are inverses of one another?

4) Which model do you think you might prefer to use? Why?

Lesson 2: Chip Models and Number Lines

Launch:

Introduce the concept of red chips and black chips. A red chip represents a negative, a black chip represents a positive. Do warm ups like "put two groups of -2 on your board" or "put 4 groups of -3 groups on your board". Introduce zero pairs, etc.. Move to more complex once they know they can start with zero pairs, like "remove2 groups of 2" What is left? (In order to remove two groups of 2 you would have to introduce zero pairs because they need to add zero pairs in to start) To model simple multiplication problems like -2 x 2 = -4. To divide chips you could introduce chip models with like 15 ÷ 5 = ? They can start with 15 black chips, split them so 5 are in a group, and count how many groups they get.

Introduce a number line to students as a way to demonstrate multiplication and division of integers. For example -5 x 2 = ? start at zero, move to the left 5, you need 2 groups so move over another 5 to the left, end up at -10. Also problems like -5 x -2 = ? start at 0, move to the left -5, you need 2 groups so move over another 5 to the left, but then it is a negative two so you take the answers opposite. The opposite of -10 is +10 so that is your answer.

The chip model/number line is to get them to understand multiplication and division of integers more concretely. The problems will not get more complicated until later when students have a better understanding of the process of multiplication and division.

Explore:

Start with a think-pair-share format, to let students get comfortable and gain confidence working with the chips/number lines. Move to small table groups of 3-4 after a few questions. While students are working on the multiplication/division problems. Challenge students to come up with a way to show the problem using both chips and number lines.

Share:

Students will share strategies on each problem. They may use magnetic chips, number lines on the whiteboard, and the document camera to share their work. Students can also share word problems or real life scenarios in which these problems could take place.

Summarize:

After walking around the room as students were working the teacher will summarize how students used chips and number lines to model multiplication and division. The teacher will ask students to explain how you could solve the problems 3 x -5 and 15 ÷ 3 using chips or a number line. The teacher will then summarize the table's ideas before heading out the door. Today's lesson was about chip models and number lines in multiplication and division, they are a powerful strategy for solving mathematical problems like this, you can use them to really understand groups of numbers. (3 groups of -5)

PROBLEMS FOR STUDENTS TO WORK ON:

1. 5 x -2 =
2. 4 x -1 =
3. -3 x -2 =
4. -5 x -2 =
5. 15 ÷3 =
6. -12 ÷2 =
   10 ÷-5 =

3.1 Introducing Multiplication of Integers

At a Glance

PACING 1 day

Mathematical Goals

- Use a number line/motion model to develop the relationship between repeated addition and multiplication with integers
- Develop and use algorithms for multiplying integers

Launch

Materials
- Transparency 3.1
- Transparency markers
- Number Lines transparency

Look at the Did You Know?

Explain the Number Relay race. Have students walk through a simulation. Ask clarifying questions to make sure students understand.

- *How far does each racer run?*
- *Explain the first leg of the relay.*

Do the first problem in Question A with them.

- *What does 5 meters per second mean?*
- *Which way is he running? Left or right? Positive or negative?*
- *What number sentence could you write to show where Hahn will be 10 seconds later?*
- *What do the 5 and the 10 stand for?*
- *Suppose Hahn were running 5 meters per second to the left. How would this change the number sentence?*
- *What do the −5, 10, and −50 indicate?*

Work in pairs and then move to groups of four for further discussion.

Explore

Materials
- Number Lines labsheet

Make sure they are writing complete mathematical sentences.

If students are struggling, summarize Question A before going on to Question B.

Summarize

Materials
- Student notebooks
- Number Lines transparency
- Labsheet 3ACE Exercise 25 parts (a) and (b)
- Labsheet 3ACE Exercise 25 parts (c) and (d)

Have students share their number sentences for Question A. Analyze how the addition and multiplication sentences are related. With part (2), ask:

- *What number sentences did you write for part (2)?*
- *What does 5×4 mean?*
- *What does multiplication mean?*

Draw students' attention to patterns with the combinations of positive and negative signs. For Question A part (3):

- *What number sentences did you write for part (3)?*
- *Here you have a positive times a negative. Why is the result negative?*
- *What does $8 \times (-6)$ mean?*

continued on next page

Summarize

continued

As students describe their number sentences, note the sign patterns.

- *In Question A part (5), you have a negative direction and a negative time. Why does it make sense that this means Tori is at a positive position on the racing field?*

Have students present their algorithms.

- *What is happening when you multiply a positive number by a negative number? A negative times a negative?*

Going Further

- *If you had a negative times a negative times a negative, would the results be negative or positive?*
- *Is multiplication commutative for negative numbers?*

3.2 Multiplication Patterns

At a Glance

PACING 1 day

Mathematical Goal

- Examine number patterns to confirm the algorithm for multiplication

Launch

Materials
- Transparency 3.2
- Transparency markers

Display the series of equations for students to observe, including the next three in the series.

- *What patterns do you notice?*
- *How do the patterns help you determine the next few equations in the series?*

Have students work in pairs to answer the questions.

Explore

As students work on the problem, listen to the patterns they are noticing.

If students struggle in Question B, suggest they think about what each sentence is saying and what seems reasonable.

Summarize

Materials
- Student notebooks

For Questions A and B, ask students to share their observations and solutions explaining why they are reasonable and how they fit the patterns they noticed.

Display both series of equations as you ask these questions.

- *If one factor is 0, what will the product be? Is this always true?*
- *The first five equations involve multiplying two positive factors and result in a positive product. Is that always true? Does a positive factor times a positive factor always give a positive product?*
- *In the second series of equations, the first five equations involve multiplying a positive factor times a negative factor and result in a negative product. Is this always true? Does a positive factor times a negative factor always give a negative product?*
- *In Question B parts (2) and (3), you multiplied two negative factors. What is the sign of the product when two negative factors are multiplied?*
- *When you multiply three numbers together, how do you look at the signs to predict the sign of the product?*

Going Further

- *Regardless of their signs, is the numerical value of the product of two numbers always the product of the two numbers? Explain.*
- *How do you find the sign of the product?*

3.3 Introducing Division of Integers

At a Glance

PACING 1 day

Mathematical Goals

- Explore division of integers using the relationship between multiplication and division found in fact families
- Recognize and solve problems involving multiplication and division of integers
- Develop algorithms for dividing integers

Launch

Materials
- Transparencies 3.3A, 3.3B
- Transparency markers

Write $36 \div 4 = 9$ on the board.

- *What does this sentence mean?*
- *How are the operations of multiplication and division related?*
- *If multiplication and division are opposite operations that undo each other, what number sentence would undo $3 \times 12 = 36$?*

Discuss the patterns in the examples in the Student Edition. Note what the signs are doing.

Have students work on the examples in the Getting Ready.

For problem 1, draw students' attention to the notion of using $48 \div 8$. Students can then use their multiplication algorithm to decide whether the 6 should be positive or negative.

Have them work in pairs and then in groups to answer the questions.

Explore

Materials
- Number Lines labsheet

As students work on the problem, listen to the patterns they are noticing. If they are struggling, you might stop them after Question B part (1), summarize what they have done so far, and then have them complete the problem.

Summarize

Materials
- Student notebooks
- Number Lines transparency

Have students display the solutions to Question A. If there are disagreements, have students discuss their solutions and explain why they make sense.

Help students generalize some rules for dividing positive and negative integers. Use questions like these as students discuss their solutions.

- *How did you decide if the quotient is positive or negative?*
- *What multiplication problems are related to this division problem?*
- *How can you use your algorithm for multiplication to decide whether the quotient is positive or negative?*

For Question B, ask students to share their answers and their algorithms for the problems in each of the three groups.

continued on next page

Summarize

continued

- *Why is the rule "a negative divided by a negative is positive" reasonable?*
- *How can you find the quotient when you divide a negative integer by a positive integer? For example, why does it make sense that* $-99 \div 11 = -9$*?*
- *How can you find the quotient when you divide a positive number by a negative number?*
- *We have found that addition and multiplication are commutative. Is division commutative?*

Name _____ Date _____ Class _____

Skill: Dividing Integers

Investigation 3

Accentuate the Negativ

Divide.

1. $14 \div 7$ **2.** $21 \div (-3)$ **3.** $-15 \div 5$

4. $-27 \div (-9)$ **5.** $45 \div (-9)$ **6.** $-42 \div 6$

7. $-105 \div (-15)$ **8.** $63 \div (-9)$ **9.** $108 \div 6$

10. $-204 \div 17$ **11.** $240 \div (-15)$ **12.** $-252 \div (-12)$

Find each product or quotient.

13. $\frac{-36}{9}$ **14.** $\frac{-52}{-4}$ **15.** $(-5) \cdot (-20)$

16. $\frac{-63}{-9}$ **17.** $(-15) \cdot (2)$ **18.** $\frac{22}{-2}$

19. $(13) \cdot (-6)$ **20.** $\frac{-100}{-5}$ **21.** $(-60) \cdot (-3)$

For Exercises 22 and 23, represent each pattern of change with an integer.

22. spends $300 in 5 days **23.** runs 800 feet in 4 minutes

24. Juan's baseball card collection was worth $800. Over the last 5 years, the collection decreased $300 in value. What integer represents the average decrease in value each year?

25. Florence purchased stock for $20 per share. After 6 days, the stock is worth $32 per share. What integer represents the average increase in stock value each day?

Launch:
Host a discussion about the class's working understanding about an algorithm that can work for division of integers. Give students time to create a multiplication table if they need, to support their work on the top portion of this skills practice without a calculator, until problems reach into 3 digits.

Explore: Modify this worksheet so that the expectation is that students not only solve problems, but create at least one additional model to represent the problem. For problems 1-12, have students choose 2 from which to create a story as well.

Share: As the class is wrapping up, have students share results from #25.

Summarize: Close the lesson with have students think-pair-share this question and then writing their own answer in their journals:

Why does it make sense that when a negative number divides a negative number, the result is positive?

(Where do we see that in a fact triangle? Chips? Number line? Or bar model?)

3.4 Playing the Integer Product Game

At a Glance

PACING 1 day

Mathematical Goal

- Develop skill at multiplying and dividing integers

Launch

Display the Integer Product Game Board and ask members of the class to explain the rules of the game. If students do not remember the rules from *Prime Time*, they can read them in the Student Edition.

Play one game with the class to be sure everyone understands how to play.

- *What factors can I put paper clips on to get a product of −12?*
- *What numbers divide −12 equally?*
- *How are these two lists related?*

Have students pair up and play the game a few times.

Materials

- Transparency 3.4
- Transparency markers
- 2 paper clips

Explore

Remind students to write down any patterns or winning strategies that they discover.

- *As you play the game, think about these questions.*

Display the following questions:

- *Is it better to go first or second? Why?*
- *What is the best move if you are second?*

As students are playing and analyzing the game, look for those who are using various strategies. Have students record mathematical sentences to show their thinking. Look for examples of students using both multiplication and division to bring out during the summary.

Materials

- Labsheet 3.4
- Paper clips (2 per pair)
- Colored pens, markers, or pencils

Summarize

Begin by discussing any strategies for playing and winning the game that students discovered.

Guide the discussion to bring out that when using positive and negative products, you have many more combinations to consider than you do when the Product Game is played with non-negative numbers. Question B can help get at this idea.

- *How would the pairs of factors in Question B compare to the list of factor pairs you could make if the game only used positive numbers?*

In Questions C and D, help students connect back to their algorithms. Writing number sentences for their verbal descriptions might be helpful.

Materials

- Student notebooks

continued on next page

Summarize

continued

Check for Understanding

Write on the overhead or board: $5 \times (-6) = -30$.

- *Use the words* factor, divisor, multiple, product, *and* divisible by, *to write four statements about the sentence* $5 \times (-6) = -30$.

Note: This lesson serves as a launch to the next as well, which is a day of practicing fluency with these concepts related to multiplying and dividing integers. The next 2 pages detail the problem set (Explore) and Share/Summarize sections to close this 2 day lesson.

Answer Questions 5–8 with "true" or "false," and *explain* your answer.

5. The sum of two negative integers is always negative.

6. The product of two negative integers is always negative.

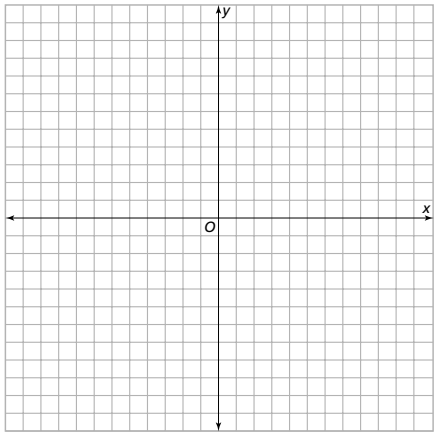
7. The sum of a negative integer and a positive integer is always positive.

8. The product of a negative integer and a positive integer is always negative.

9. One integer added to another integer gives a sum of $^{-}9$. When the smaller integer is subtracted from the greater integer, the difference is 1. What could the two integers be?

10. a. Below is a grid with four quadrants. Plot the following points, and connect them with line segments.

Point A $(1, 0)$ Point B $(3, 4)$ Point C $(4, 0)$



b. On the same grid paper, transform your figure ABC using the rule $(2x, 2y)$.

c. On the same grid paper, transform your figure ABC using the rule $(^{-}2x, ^{-}2y)$.

d. On the same grid paper, transform your figure ABC using the rule $(^{-}2x, 2y)$.

e. On the same grid paper, transform your figure ABC using the rule $(2x, ^{-}2y)$.

f. Without drawing, predict what will happen to ABC using the rule $(3x, 3y)$.

g. Without drawing, predict what will happen to ABC using the rule $(^{-}3x, ^{-}3y)$.

h. Without drawing, predict what will happen to ABC using the rule $(^{-}3x, 3y)$.

11. Rewrite these temperature readings from lowest to highest.

$^{-}9°$ $14°$ $^{-}2°$ $0°$ $8°$ $^{-}1°$ $1°$

12. a. Suppose the temperature is 6°. What will the temperature be if it rises 22°?

b. Suppose the temperature is 6°. What will the temperature be if it falls 7°?

13. a. Suppose the temperature is $^{-}6°$ (6° below 0°). What will the temperature be if it rises 13°?

b. Suppose the temperature is $^{-}6°$. What will the temperature be if it falls 15°?

14. Barry plays fullback on his high school football team. Sometimes he gains yardage ($^{+}5$ means a 5-yard gain). Sometimes he loses yardage ($^{-}3$ means a 3-yard loss). Determine Barry's total yardage in each game below.

a. Game 1: $^{+}4$ $^{+}6$ $^{+}7$ $^{+}1$ $^{-}8$

b. Game 2: $^{+}6$ $^{-}3$ 0 $^{+}15$ $^{-}1$ $^{+}8$ $^{+}11$ $^{-}6$

15. Suppose the Rocky Mountains have 72 centimeters of snow. Warmer weather is melting the snow at a rate of 5.8 centimeters a day. If the snow continues to melt at this rate, after seven days of warm weather, how much snow will be left?

16. Write a number less than $^{-}1000$.

Question Bank *(continued)*

A

17. After several minutes of playing MathMania, three teams have the following scores:

SuperSmarties	DynaBrains	MegaMinds
650	$^{-}150$	200

a. The SuperSmarties are how many points ahead of MegaMinds? Write a number sentence that could be used to find this amount.

b. The SuperSmarties are how many points ahead of DynaBrains? Write a number sentence that could be used to find this amount.

c. The MegaMinds are how many points ahead of DynaBrains? Write a number sentence that could be used to find this amount.

Tell how far apart the two numbers are on a number line.

18. $^{-}15$ and $^{+}20$ **19.** 37 and 17 **20.** $^{-}5$ and $^{+}12$

Solve the problem.

21. $18 - 27 = \square$ **22.** $27 - 18 = \square$

23. $14 - {}^{-}8 = \square$ **24.** $^{-}14 - {}^{+}8 = \square$

25. $^{-}150 - {}^{+}24 = \square$ **26.** $90 - {}^{-}99 = \square$

27. $16 + 12 + {}^{-}4 = \square$ **28.** $\square + 21 = 13$

29. The temperature for the past 8 hours has been changing at the rate of $^{-}1$ each hour. The meteorologist predicts that the temperature will continue changing like this for the next 6 hours. The present reading is 0°.

a. What was the temperature reading 7 hours ago?

b. What temperature is predicted for 6 hours from now?

c. When was the temperature reading 6°?

d. When is the temperature expected to be $^{-}8$°?

Share: Have students share solutions to #5-8 on the first page of practice.

Summarize: Have students rewrite their thinking in their journals to answer 5-8 with their new insight from the discussion.

Ex: Students who were already on track can enhance their answers with clearer language choices. Students who missed the mark can rewrite their thinking to make it accurate.

4.1 Order of Operations

At a Glance

PACING 1 day

Mathematical Goal

- Explore the use of the order of operations to order computation in problems

Launch

Use the Getting Ready to engage the class in what the challenge of the problem will be. Have the students look at the problem and make their own predictions about which should be correct. Then turn to a discussion of the rules for the order of operations and the examples given. Return to the Getting Ready as an example and decide with the class what the answer is according to the agreed-upon rules of order.

Remind students to go back to these rules as needed throughout the problem.

Talk about the use of parentheses. Make sure that they understand parentheses as a grouping symbol that indicates that what is in the parentheses is to be treated as a single entity. You need to compute what is in the parentheses first. Also, you can insert parentheses to make sure the expressions you write reflect the order of operations you intend. Review what exponential notation means using examples like: $3^2 = 3 \times 3$, $2^3 = 2 \times 2 \times 2$ and $2^4 = 2 \times 2 \times 2 \times 2$.

Think-Pair-Share is a good classroom arrangement for this problem.

Materials

- Transparencies 4.1A, 4.1B
- Transparency markers

Vocabulary

- order of operations

Explore

Ask students to say in words how the mathematical sentences they write or have to interpret should be computed.

For Question C, suggest they use the order of operations rules to find an answer. Then think about which operation can make an answer greater.

When most students have completed at least one problem in Question C, begin the whole-class summary.

Summarize

Go over Question A and use the discussion to summarize the strategies students have used to help them both write and interpret mathematical sentences. Have students say in words how the expressions should be computed.

Question B provides practice in using the rules for the order of operations.

For Question C, ask students to share strategies that helped them use parentheses to make answers less and strategies that helped make answers greater.

Materials

- Student notebooks

continued on next page

Summarize

continued

Question D is a challenge because of its length and complexity. If all students have not started this problem, give them a few minutes now to work on it before discussing it.

Have students display their thinking and discuss the problem in steps so that students can reason through and apply the order of operations when the string of symbols is long.

Check for Understanding

For each example, tell the sequence of computations needed to get the correct answer and give the answer.

1. $2^2 + 7 \times (-3) - 5$

2. $(2^2 + 7) \times (-3) - 5$

3. $(2^2 + 7) \times (-3 - 5)$

4.2 Distributing Operations

At a Glance

PACING 2 days

Mathematical Goals

- Model the Distributive Property with areas of rectangles that have edges subdivided
- Develop and use the Distributive Property of multiplication over addition

Launch

Materials
- Transparencies 4.2A, 4.2B

Draw a picture of a 6 meter × 10 meter rectangle on the board. Indicate that this represents the area of a back yard where the landowner has marked off a garden across the 10-meter side that is 2 meters long.

- *What is the area of the entire back yard?*
- *What is the area of the garden?*
- *What is the area of the remaining yard without the garden?*
- *How do these parts relate?*
- *Which is easier to compute: 6×10 or $(6 \times 2) + (6 \times 8)$?*
- *Why would you write $(6 \times 2) + (6 \times 8)$?*
- *The questions you will answer in this problem are like the problem you just analyzed. You will be trying to find expressions for the area of a rectangle and its parts that make computing or subdividing the area easy.*

If your class period is not a full hour, assign and summarize Questions A–D in class. Assign Question E as homework and discuss it the next class day.

Have students work in small groups on this problem.

Explore

Continue to ask students to say in words what the sequence of the computations is in each of the number sentences they write.

Try to visit each group for some part of Question A or B so that you can give help, if needed.

Question C asks the reverse of Questions A and B but uses what is learned in Questions A and B. If students are struggling with Question C, ask:

- *What are the dimensions of the needed rectangle?*
- *How do you show that one side is divided?*

Summarize

Materials
- Student notebooks

Discuss each of the parts with your students. Look carefully at the rectangle model and make sure they understand how this model can support their thinking. Have students sketch the models on the board or overhead and write the areas on each of the sections.

continued on next page

Summarize

continued

For Question D, if students struggle, replace the x with a box or a number; work through the problem, then replace the box or the number with an x.

- *What do these problems show us about computing the area of rectangles?*

Going Further

- *Write expressions to show two different ways to represent the area of this rectangle. (Draw a rectangle with the dimensions: $3 + x$ by $5 + x$.)*

4.3 The Distributive Property and Subtraction

At a Glance

PACING 1 day

Mathematical Goals

- Develop and use the Distributive Property of multiplication over subtraction
- Use the Distributive Property to solve problems

Launch

Use the Getting Ready to review and summarize the Distributive Property for multiplication and addition. Relate factoring to writing the expression as a product of factors and expanding the expression to writing the expression as a sum of addends.

Raise the issue of whether the property would hold if the operations were multiplication and subtraction. Leave the questions for students to ask themselves as they work on the problem.

- *Do you think the Distributive Property can be used to expand or factor an expression with subtraction? Why or why not?*
- *Can you give an example?*
- *When you have a subtraction expression, can you use integers to write the expression as an addition expression?*

Have students work in pairs.

Materials
- Transparency 4.3
- Transparency markers

Vocabulary
- Distributive Property

Explore

If students are having trouble with subtraction, give them a simple problem like $2 - 3$, and ask them to write an addition problem with the same answer.

Also suggest that they apply the Distributive Property and check to see if the two forms for the expressions give the same answers.

- *What would the expression $-5 \times (3 - 2)$ look like if you expanded it using the Distributive Property?*
- *Does this give the same answer as the original expression?*

Summarize

Have students report on Questions A–C. Continue to ask questions like:

- *How are addition and subtraction related?*
- *Give an example where this idea is useful.*
- *How can you tell factored form from expanded form?*
- *Which form seems the most useful for computing, expanded or factored form?*
- *In $-5 \times (3 + 2)$, how can you tell that -5 and $(3 + 2)$ are factors?*

Materials
- Student notebooks

continued on next page

Summarize

continued

- *Explain how you can use the Distributive Property to expand an expression that is the product of two factors where one is a sum (or difference) of two numbers.*
- *Explain how you can use the Distributive Property to factor an expression that is the sum of two terms with a common factor.*

Questions D and E are very important. Call on more than one student to present a solution and to describe their strategy for solving the problem.

Review the notes on notation with students.

Lesson 13: Multiplying/Dividing/Order of Operations Practice

Launch:

Teacher will introduce real-life scenarios that involve multiplying/dividing integers, along with practice for understanding distributive/commutative properties, and order of operations.

Explore:

Students will work in groups in a Think-Pair-Share model for the first two questions. They will then work in their table groups on the remainder of the questions. When students are working walk around and ask things like "What does it mean when an operation is commutative?", "What does it mean to distribute?", and "Why are the order of operations important to understand?"

Share:

Students will Think-Pair-Share concepts they find most important to remember in multiplying/dividing integers, distributive/commutative properties, and order of operations. (Commutative only holds true for multiplication and addition….etc..) While students are sharing teacher will write ideas on the board. Basically at this point the teacher wants them to share what they have learned so far about multiplying/dividing/commutative/distributive/order of operations of integers.

Summarize:

Teacher will summarize student strategies. Stressing the importance in knowing which properties (commutative and distributive) hold true for which operations. Knowing the properties can help you rewrite a math problem successfully.

Name: _____

STUDENT WORK PROBLEMS

Answer the following questions with True or False and ***explain*** your answer.

1. The product of two negative integers is always negative.

2. The product of a negative integer and a positive integer is always negative

3. A pizza place has 282 pounds of pizza dough. It only uses 1.1 pounds per pizza. How many pizzas can the pizza place make?

4. The following list shows the yards gained and lost on each play by the Mathville Mudhens in the fourth quarter of their last football game:

-8, 20, 3, 7, -15, 4, -12, 32, 5, 1

Write an expression that shows how to compute their average gain or loss per play. Then compute the average.

5. Find the missing part(s) to make the expression true.

$5(9 + 6) = 5(\square) + 5(\square)$

$\square(3.8) = 9(4) - 9(\square)$

6. Copy and place parenthesis to make each statement true.

$6 + 6 \div 6 \times 6 + 6 = 24$

$6 - 6 \div 6 \times 6 + 6 = 0$

7. Write a story problem that is answered by finding the value of *n*.

$7n = -42$

Lesson 14: Dealing Down

Launch:

Teacher will introduce the game Dealing Down to the students. Students will follow along with the rules. Shuffle 25 cards, deal four cards to the center, each player writes an expression with the least possible quantity. Players compare answers and discuss how they know their quantity is accurate and the least possible. Each player with the lowest expression gets 1 point. Record the results in a table and play more rounds. Let students know that as they are playing they should be thinking about their strategy (including order of operations, commutative, distributive properties, and operating with positive and negative numbers). Model a round for the students.

Explore:

Start with a practice round to make sure everyone understands the game. Once the practice round is complete they may start keeping score. Students may play in their table groups. Walk around and make sure students are participating and ask questions like "Why did you decide to group these numbers together?" "How would dividing help you get a smaller solution" "Would your answer change if I multiplied by ….."

Share:

Tables will share their smallest solution to the class. Once tables are finished individual students may share their smallest solution. As they are sharing hint to things that help them divulge a little of their strategy (Don't turn it into a complete strategy talk though---as they will talk about strategy in Lesson 15) They may use the whiteboard and the document camera to share their work.

Summarize:

After walking around the room as students were working the teacher will summarize how students used order of operations, distributive property, commutative property, and operations with integers to achieve the smallest sum. Today's lesson was about using properties of integers, operations with integers, and order of operations to change the value of a group of numbers. Knowing these things can be a powerful strategy in mathematics.

DEALING DOWN

Dealing down is a mathematical card game that tests your creative skill at writing expressions. Play several rounds of the game. Then write a report on the strategies you found.

How to Play Dealing Down

- Work in small groups.
- Shuffle the 25 cards.
- Deal four cards to the center of the table.
- All players use the four numbers to write an expression with the least possible quantity.
- Players compare answers and discuss how they know their quantity is accurate and the least possible.
- Each player with an expression for the least quantity gets 1 point.
- Record the results of that round in atable like the one on the following page and play more rounds.
- The player with the most points at the end of the game wins.

Score Sheet for Dealing Down

Round 1

Cards Dealt	Expression With the Least Quantity	Who Scored a Point
Why That Expression Has the Least Quantity:		

Round 2

Cards Dealt	Expression With the Least Quantity	Who Scored a Point
Why That Expression Has the Least Quantity:		

Round 3

Cards Dealt	Expression With the Least Quantity	Who Scored a Point
Why That Expression Has the Least Quantity:		

Round 4

Cards Dealt	Expression With the Least Quantity	Who Scored a Point
Why That Expression Has the Least Quantity:		

Round 5

Cards Dealt	Expression With the Least Quantity	Who Scored a Point
Why That Expression Has the Least Quantity:		

Dealing Down Cards

0	-1	-2	-3	-4
-5	-6	-7	-8	-9
$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0.5	$\frac{1}{3}$
0.25	1	10	5	7
8	2	3	4	-10

Lesson 15: Reflection

Launch:

Teacher will review the game played in Lesson 14 Dealing Down. Teacher will make a statement like "_____ won in group 2, does this mean he is good at guessing where the numbers should go?" Inevitably when working with 7th graders that student will speak up to share a strategy. Others may agree or disagree. Talk about strategy with the kids and have them share their strategies briefly. The assignment will be to write a reflection about strategies for writing the least possible quantity using four numbers. (Go over rubric with students so they know what to expect)

Explore:

Have students play another 2 rounds of Dealing Down so that if a student didn't have a strategy the first go around they have had a chance to hear some options and test.

Students will have time to work on their reflection. They may type it in the classroom blog.

Share:

Students will summarize their reflection to the class, giving each student a chance to get a snapshot of their strategies. Students will then go to the classroom blog and share comments on their classroom blogs. (Students know at this point that comments like "good job!" are not as helpful as "I agree with your idea that dividing can be helpful when finding a small solution!")

Summarize:

Teacher will summarize student reflections. Stressing the importance of the following ideas in each students strategy.

- Operating with negative and positive numbers
- Order of operations (including use of parenthesis and exponents)
- Commutative Property of Addition and Multiplication
- Distributive Property

RUBRIC FOR REFLECTION

	Exemplary 4 Complete -Shows understanding of mathematical concepts & procedures. -Complete, clear, coherent explanation	**Proficient 3 Reasonably Complete** -Shows understanding of *most* mathematical concepts & procedures. -Lacks detail of clarity.	**Basic 2 Partial** -Shows *some* undestanding of mathematical concepts & procedures. -Explanation is unclear or lacks detail.	**Novice 1 Inadequate** -Shows *little* understanding of mathematical concepts & procedures. -Explanation is insufficient or not understandable.	**No Attempt 0**
Operations with Positive and Negative Numbers					
Order of Operations					
Commutative Property					
Distributive Property					
Explained Strategies					
Criteria and Procedures					

Name:_____

Post-test – Number & Operation Unit

1. Explain why these results are the solutions to these equations.

a. 13 * -7 = -91

b. -8 * -20 = 160

c. 99/-3 = -33

d. -36/-12 = 3

e. 0/18 = 0

f. $\frac{1}{3} \times \frac{-5}{7} =$ **-6/21**

2. **Each bottle rocket takes 7.2 oz of gunpowder. A fireworks factory has 236 pounds of gunpowder on hand after this year's 4th of July. Write a number sentence showing how you would figure out how many bottle rockets they can make for next year?** (Hint: There are 16 oz in 1 pound)

3. **Josie owes her broth $56. She had owed more, but she's been paying $8 a month on her debt for the last five months.**
   a. **How much was Josie in debt 7 months ago?**

   b. **At the present rate, how much longer will it take Josie to pay off her debt? Explain your reasoning.**

4. For the 3 problems below, model each situation and explain why the circled letter is the result of the equation.

$^{-}8 \times {}^{-}29 =$

A. 232 **B.** $^{-}232$ **C.** 211 **D.** $^{-}211$

$^{-}23 \div 0.5 =$

F. $^{-}11.5$ **G.** $^{-}46$ **H.** 11.5 **J.** 46

$45 - 23 \times 2 + 1 =$

A. 43 **B.** $^{-}22$ **C.** 0 **D.** 68

5. **Model or write a story for a problem that is answered by finding the value of n.**

-8n = -32

6. **Jason wants to take 5 of his friends to a movie. He knws it is $5.50 for a ticket and $3.25 for popcorn. How much will it cost if he past for the movie and popcorn for all 6 people? Show your result in two different ways, with one being an equation.**

7. **Given the expression 8(15-7), circle the expression that is not equivalent.**

   a) **(8*15)-7**

   b) **8*8**

   c) **(8*15) – (8*7)**

   d) **(8*15)+(8*-7)**

9. **Create a problem that shows how the Distributive Property is like finding the area of a rectangle.**

10. **Place these rational numbers in order from least to greatest.**

-23, -1/2 , 0.85, -0.198, 56, 1/100

12. **Insert parentheses where needed to show how to get this result.**

1 + -7 * -3 = 22